DUSO Mathematics League 2015 - 2016

Sectional Championship   **Calculators are not permitted on this contest.**

Part I.   **ALGEBRA I**   *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

S - 1. Suppose that for some sequence u_n, $u_0 = 3$ and $u_{n+1} = -3u_n + 7$. Compute u_2.

S - 2. Factor completely over the integers: $x^4 - 25x^2 + 40x - 16$

Part II.   **GEOMETRY**   *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

S - 3. Square $ABCD$ has area 25. If square $ABCD$ is revolved $360°$ about side $\overline{AB}$, a solid is formed. Compute the volume of the solid.

S - 4. The line $y = 2x + 2$ is the image of the line $y = \dfrac{1}{2}x - 4$ after a reflection in the line $y = ax - b$ for some positive integers a and b. Compute the ordered pair (a, b).

Part III.   **ALGEBRA II / ADVANCED TOPICS**   *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

S - 5. Compute all values of x that satisfy the following: $\dfrac{x^3 - x^2 - 4x + 4}{x^2 - 3x + 2} = 0$

S - 6. The digits $1, 1, 2, 2, 3, 3, 4, 4$ are used to make an 8-digit number. Compute the probability that in the 8-digit number, the 1's are next to each other and so are the 2's and 3's and 4's.

R-1. Compute the least positive integer value of x that satisfies $x^2 - 6.1x - 7.92 > 0$.

R-2. *Let N be the number you will receive.* A goat is tethered to the corner of a rectangular barn whose length is 10 meters and whose width is 4 meters. The tether is N meters long. Compute the goat's grazing area in square meters.

R-3. *Let N be the number you will receive.* The circle centered at the origin with area N passes through two lattice points in the first quadrant: (A, B) and (B, A) where $A < B$. Pass back the ordered pair (A, B).

R-4. *Let (A, B) be the coordinates you will receive.* The graph of the equation $y = A\cos x + B$ has a minimum at (C, D) where $0 < C \leq 2\pi$. Compute $C + D$.

R-5. *Let N be the number you will receive.* Circle O has diameter $\overline{AB}$. A circle P is inscribed in one of the semicircles formed by $\overline{AB}$. The semicircle has perimeter N. Compute the area of the inscribed circle.